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LETTER TO THE EDITOR

Universal distance ratios for 2D SAWs: series results

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Abstract. The mean-square distance of a monomer to the origin and the mean-square distance of the centre of mass to the origin are studied by exact enumeration on the square and triangular lattices. The numerical results are inconsistent with the theoretical prediction of Cardy and Saleur by more than fifteen times the numerical uncertainty in our calculation.

Due to their connection with polymer physics and the $n \to 0$ n-vector model [1], self-avoiding walks (SAWs) have attracted much interest over the years. One aspect of the metric properties of this model has recently been studied by Cardy and Saleur [2]. Defining

$$\begin{aligned} \nu_N &= \langle R_g^2 \rangle_N / \langle R_e^2 \rangle_N \\ \mu_N &= \langle R_m^2 \rangle_N / \langle R_e^2 \rangle_N \\ F_N(y_t/y_h, \nu_N, \mu_N) &= \left(2 + \frac{y_t}{y_h}\right)\nu_N + \frac{1}{8} - \mu_N \end{aligned} \tag{1}$$

using the c-theorem of Zamolodchikov [3], Cardy *et al* [2] derived a universal relation for SAWs on two-dimensional lattices:

$$\lim_{N\to\infty} F_N(y_t/y_h, \nu_N, \mu_N) = 0 \tag{2}$$

Here $\langle R_g^2 \rangle$ and $\langle R_e^2 \rangle$ are the usual mean-square radius of gyration and the mean-square end-to-end distance respectively, while $\langle R_m^2 \rangle$ is the mean-square distance of a monomer to the origin. $\langle \ldots \rangle_N$ denotes the average over the ensemble of N-step SAWs. The exponents y_t and y_h are known from Coulomb gas or conformal invariance techniques [4] to be $\frac{4}{3}$ and $\frac{91}{48}$ respectively for 2D SAWs. The second-order ε expansions [5, 6] predicted $\nu_\infty = 0.1428$ and $\mu_\infty = 0.4367$. The substitution of the above data into (2) gives $F_\infty = 0.074$. Data from an early exact enumeration calculation [7] yield a similar result with $F_\infty = 0.066 \pm 0.004$.

The quantities $\langle R_g^2 \rangle$ and $\langle R_e^2 \rangle$ have been studied extensively by many authors. $\langle R_g^2 \rangle$ is known to 21 terms on the square lattice and to 15 terms on the triangular lattice [8], while $\langle R_e^2 \rangle$ is known to 27 terms on the square lattice [9-13] and to 19 terms on the triangular lattice [10-13]. However, it appears that there have been no studies of $\langle R_m^2 \rangle$ since the 20 year old study by Domb and Hioe [7]. The present letter is aimed at an exact enumeration study of $\langle R_m^2 \rangle$ on the square and triangular lattices to compare with the result of Cardy and Saleur.

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The N-step self-avoiding walk configurations are generated by a conventional backtracking method and the square distance of every monomer to one of the end points is accumulated, giving $(N+1)C_N\langle R_m^2\rangle_N$. The calculated data for the square and triangular lattices are listed in table 1. For every SAW configuration, we also summed the vector distance of every monomer to the origin, then accumulated the square of this quantity over all the SAW configurations. This gives us $(N+1)^2C_N\langle R_c^2\rangle_N$. Here $\langle R_c^2\rangle$ is the mean-square distance of the centre of mass to the origin (table 1). These data confirm the relation $\langle R_m^2\rangle_N = \langle R_g^2\rangle_N + \langle R_c^2\rangle_N$ and show that our definition is consistent with previously published data. Using the known data [8-13] for $\langle R_g^2\rangle$ and $\langle R_e^2\rangle$, and our data for $\langle R_m^2\rangle$, as tabulated in table 1, we calculated the ratios ν_N and μ_N and extrapolated these sequences by a variety of sequence extrapolation techniques. These included all the methods that have been found [14] to be most successful for lattice statistics problems. These included Nevelle tables, Levin's u-transform, Brezinski's θ algorithm and Wynn's ε algorithm. The results were consistent from method to method and for both lattices. We found that $\nu_\infty = 0.1396 \pm 0.001$ and $\mu_\infty = 0.4375 \pm 0.002$ included estimates from all methods, for both lattices. The quoted error bars are in fact more than double the actual spread in the estimates. Combining these results (2), we obtain $F_\infty = 0.065 \pm 0.004$, in complete agreement with earlier, shorter, enumerations.

This discrepancy with the theoretical calculation of Cardy and Saleur means that at least one of the results is wrong. In such circumstances it is natural to suspect the numerical results before the theoretical calculation. It is possible that we are seeing a 'short series' effect, but the precise agreement of the corresponding calculation for

Table 1. Exact enumeration results for the mean-square distance of the monomers to the origin and the centre of mass to the origin on two-dimensional lattices.

	Square lattice		Triangular lattice	
N	$\frac{1}{4}(N+1)C_N\langle R_m^2\rangle_N$	$\frac{1}{4}(N+1)^2C_N\langle R_c^2\rangle_N$	$\frac{1}{6}(N+1)C_N\langle R_m^2\rangle_N$	$\frac{1}{6}(N+1)^2C_N\langle R_c^2\rangle_N$
1	1	1	1	1
2	11	19	17	29
3	74	180	178	430
4	390	1 228	1 476	4 602
5	1 801	6 919	10 667	40 697
6	7 537	34 251	70 359	317 319
7	29 684	155 312	434 708	2 261 924
8	110 796	656 984	2 557 166	15 075 338
9	399 375	2 642 195	14 477 823	95 355 739
10	1 391 809	10 173 705	79 492 861	578 428 105
11	4 741 466	37 908 012	425 633 898	3 390 550 172
12	15 783 154	137 098 900	2 231 674 940	19 314 366 036
13	51 704 949	484 531 231	11 494 836 257	107 394 577 387
14	166 550 157	1 675 623 115	58 310 378 811	584 885 810 177
15	530 165 200	5 696 578 328	291 901 836 462	3128 553 102 050
16	1 666 083 296	19 048 928 344		
17	5 188 200 085	62 865 722 893		
18	15 993 447 527	204 788 961 203		
19	48 946 213 794	660 181 353 644		
20	148 574 713 674	2105 992 726 468		
21	448 343 690 109	6661 347 725 003		

two different lattices makes this, in our view, unlikely. Series of similar, indeed shorter, length have in the past agreed with analogous theoretical calculations. We therefore consider it appropriate to bring into question the theoretical calculation.

In summary, we have calculated the mean-square distance of monomers to the origin on the square and triangular lattices respectively. Combining these data with the known data for the radius of gyration and the mean-square end-to-end distance, we compared the numerical calculation with the theoretical prediction of Cardy and Saleur. The numerical results are in disagreement with the above theoretical prediction within the quoted errors. We also calculated the mean-square distance of the centre of mass to the origin to ensure that the definition we are using is compatible with the available data.

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